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THE SOLUTION OF PROBLEMS OF MATHEMATICAL LEARNING THEORY USING COMPUTER MODELS

Abstract
At computer modeling of process of training it is usually supposed that all elements of learning material are forgotten with an identical speed. But in practice that knowledge which are included in educational activity of the pupil are remembered much more strongly and forgotten more slowly then knowledge which he doesn't use. In article are analyzed: 1) the model, which takes into account the reduction of coefficient of forgetting for issues included in the activities of the pupil; 2) the model considers probability of the appeal to issues from previous themes of educational course; 3) the model, which takes into account that on lesson occurs transition of weak knowledge to the category of strong knowledge. The computer model is considered, programs in the Pascal are submitted, results of modeling are given and analyzed.

Keywords
computer modeling, didactics, education system, information approach, learning, training, simulations

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1. Statement of a problem. The Mathematical Theory of Learning is direction of didactics in which for research of system “teacher-pupils” are used mathematical methods (Atanov, Pustynnikova, 2002; Leont’ev, Gohman, 1984; Novikov, 1998). Development of information technologies created prerequisites for use of a method of imitating modeling for a research of didactic processes. Its essence consists that the real pedagogical system is replaced with abstract model, - some idealized object which behaves like the studied system. Such model can be system of logical rules, the mathematical equations or the computer program allowing to make a series of experiments at various parameters, entry conditions and external influences. If we are changing initial data and parameters of model, then it is possible to investigate ways of development of system and to define its state at the end of training.

Further development of didactics is connected with use of a method of imitating modeling for the solution of set of tasks corresponding to these or those situations arising in the course of training (Mayer, 2014). All problems of Mathematical Theory of Training can be divided into two classes: 1) the predictive: knowing parameters of pupils, characteristics of the used methods and the training program (distribution of educational information), to determine their level of knowledge (or formation of skill) during and at the end of training; 2) the optimizing: to find an optimum way of training (the applied methods, duration of lessons, etc.) at which the level of knowledge of trainees will reach demanded (or maximum) value at set (or minimum) expenses of the teacher and pupils. The solution of each task supposes: 1) mathematically strict formulation of a condition (pupil's parameters, influence of the teacher, duration of lesson, etc.) ; 2) choice of
mathematical model; 3) creation of the computer program which is modeling behavior of the investigated didactic system; 4) realization of series of computing experiments; 5) interpretation and analysis of results.

Now the discrete and continuous models of training which are based on automatic approach and the solution of the differential equations [4 - 8] are known. In certain cases use multi-agent modeling at which each pupil is replaced with the program agent functioning irrespective of other agents (Ivashkin, Nazojkin, 2011). Also there are imitating models using Petri's networks, genetic algorithms, matrix modeling. All this simulations have common fault: they don't consider that the elements of a learning material (ELM-s) acquires and remembers by the pupil aren't equal. Those ELM-s (knowledge, issues, notions) which are included in activity of the pupil, turn into strong knowledge and forget more slowly; that ELM-s which aren't included - forget quicker. In the course of educational activity weak knowledge gradually becomes strong. The problem consists in creating the imitating model of learning which is considering distinction in speeds of forgetting of various ELM-s and transition of weak knowledge to the category of strong knowledge. We made the assumption: computer imitation will correspond to real process of training more precisely if to consider that: 1) durability of assimilation of various ELM-s isn't identical therefore all ELM-s should be divided into several categories; 2) strong knowledge is forgotten significantly more slowly than the weak; 3) weak knowledge at their use by the pupil gradually becomes strong.

2. Training as a result of multiple repetition of one ELM at a lesson. We will consider the pupil who in the course of training must to solve sequence of the certain problems of the same theme. For example, during a lesson he has to sum numbers (or to read separate words, to perform tasks of test) in certain timepoints. The rest of the time at a lesson he is engaged in other educational activity which doesn't interest us. Let at the moment $t_i$ the pupil start solving a problem in $i$-th time, thus the level of assimilation of the corresponding ELM at it increases to $Z=1$. We will consider that time of the solution of a task $\tau$ (or time spent for work with this ELM), depends on that, how many time $s$ this problem was solved earlier. It is possible to assume that with growth $s$ time $\tau$ decreases under the law: $\tau = 1 + 1.5e^{-s/5}$ the conventional units of time (CUT), aspiring to $\tau_{\infty} = 1$ CUT. Having performed a task and having increased the level of knowledge of the corresponding ELM to 1, the pupil switches to the solution of other educational task and starts forgetting the acquired ELM according to the law of forgetting $dZ/dt = -\gamma \cdot Z$ . We will recognize that at increase in number of use $s$ of this ELM it is remembered better. The coefficient of forgetting of this ELM decreases, for example, under such law: $\gamma = 0.002e^{-s}$ (CUT$^{-1}$). The text of the computer program 1 in the environment of Free Pascal which models training when using ELM in timepoints 3, 6, 9, 12, 15, 18 CUT is given below.

![Figure 1](image-url)

**FIGURE 1. RESULTS OF MODELING: 1) CHANGE OF LEVEL OF KNOWLEDGE OF ONE ELM AS A RESULT OF 6 REPETITIONS; 2) STUDYING 10 ELM-S DURING T=300 CUT.**
Results of modeling are presented in fig. 1.1. It is visible that after the first and second appeal to this ELM the acquired knowledge quickly is forgotten, and after the fifth and sixth is forgotten very slowly. In result of repeated use of this ELM the coefficient of forgetting decreases practically to 0, information is strongly remembered.

2. Training as a result of multiple repetition of a set of ELM-s at one lesson. Now create a model of studying of \( N \) ELM-s during of a lesson. For example, the pupil studies new \( N \) words of a foreign language which are numbered from 0 to \( N \). Reading the text, the pupil at the moment \( t_1 \) meets the word 2 and during time \( \tau_2 \) translates it, at the moment \( t_2 \) meets the word 5 and during time \( \tau_5 \) transfers it, to the moment \( t_3 \) - with the word 1, etc. When the pupil translates \( i \)-th the word the first time (\( s_i = 1 \)), he addresses to the dictionary and writes out a word meaning, the second time - looks in a notebook, the third time - translates on memory, etc., each time spending smaller time \( \tau_i \). We will consider that these ELM-s in a random way meet to the pupil and with \( i \)-th ELM he spends \( \tau_i = 1 + 2e^{-s_{i}/2} \) CUT (time of the solution of a task) for work, where \( s_i \) - number of appealing. In process of increase \( s_i \) there is a reduction of coefficient of forgetting \( i \)-th ELM under the law \( \gamma_i = 0.002e^{-s_{i}/3} \) CUT\(^{-1}\).

The program 2 is used; it builds schedules: 1) dependences of quantity of total knowledge \( Z \) on time; 2) the average time \( \tau \) of the solution of a task for all ELM-s from time; 3) average coefficient of forgetting \( \gamma \) for all ELM-s from time. The turning-out curves at \( N = 10 \) and \( T = 300 \) CUT are represented in fig. 1.2. It is visible that during training the total level of knowledge on average raises, the average time \( \tau \) of the solution of a task decreases, aspiring to the limit \( \tau_\infty \), the average coefficient of forgetting \( \gamma \) decreases, aspiring to zero.

Program 2.

```
Program Study_N_ELM; { Free Pascal }
{N+}Uses crt, graph; Const N=13; dt=0.005; Mt=1;
Var t,tt,g,t2,SZ,SZ1,ST,Sg: single;
i,j,Gd,Gm: integer; s: array[1..N] of integer;
Z: array[1..N] of single;
BEGIN Gd:= Detect; InitGraph(Gd,Gm,'c:\bp\bgi');
Randomize; t:=3;
Repeat t:=t+dt; tt:=tt-dt;
For i:=1 to N do if i<>j then Z[i]:=Z[i]-2e-3*exp(-s[i]/3)*Z[i];
If (tt<=0)and(t>50)and(t<350) then begin
  j:=round(random(N*10)/10)+1;
  \[ \]
END.
```
If \( j > N \) then \( j := 1; \ Z[j] := 1; \ inc(s[j]); \)
\( tt := 1 + 2 \exp(-s[j]/2); \ t2 := tt; \) end;
\( SZ := 0; \) For \( i := 1 \) to \( N \) do \( SZ := SZ + Z[i]; \ ST := 0; \)
For \( i := 1 \) to \( N \) do \( St := St + (1 + 2 \exp(-s[i]/2))/N; \ Sg := 0; \)
If \( t < 350 \) then line(10 + round(Mt*t), 480-round(30*SZ),
\( 10 + \) round(Mt*t2), 480-round(30*SZ1));
If \( t < 350 \) then circle(10 + round(Mt*t), 485, 1);
circle(10 + round(Mt*t), 480-round(8E+4*Sg), 1);
circle(10 + round(Mt*t), 480-round(80*ST), 1);
circle(10 + round(Mt*t), 480-round(30*SZ), 1);
circle(10 + round(Mt*t), 480, 1); \( SZ1 := SZ; \)
circle(10 + round(Mt*t), 200-round(100*Z[2]), 1);
until (KeyPressed) \{ or(t>700); \}
\{ writeln(t, ’ Znaniya ’, SZ, ’ zabivan ’, Sg); \ readkey; \} CloseGraph;
END.

Results of modeling for \( N = 6 \) and 10 at identical durations of a lesson are
presented in fig. 2.1. If quantities \( N \) of the studied ELM-s increases, the numbers \( s_i \) of
appeals to each ELM decreases therefore they are remembered less strongly. As a result
average coefficient of a forgetting \( \gamma \) at the end of training the too big. Therefore the
level of knowledge after the end of training decreases owing to forgetting, and at \( N = 6 \)
remains almost constant.

![Figure 2. Results of Modeling: 1) Studying 6 and 10 ELM-s; 2) Studying 20 ELM-s; 3) Training at a Lesson Lasting \( T = 300 \) Cut at Various Quantity of ELM-s.](image)

At further increase in quantity of the studied ELM-s the curve after the end of training
quickly decreases. The result of modeling at \( N = 20 \) is given in fig. 2.2. So, the analyzed
model shows that the quantity \( N \) of ELM-s studied at one lesson shouldn’t be too great.
At big \( N \) pupil acquires knowledge worse and then it is quickly forgotten.

Now we will study dependence of level of knowledge \( Z \) of the pupil on number \( N \)
of the studied ELM-s (which proportional to speeds of transfer of educational information \( \nu = N / T \)). We will change program 2 so that the studied ELM-s would follow one after
another, but not in a random way, and we will calculate the level of knowledge of the
pupil $Z$ and average coefficient of forgetting $\gamma$ at the $N = 3$. After that we will repeat calculations at $N = 5, 8, 11, \ldots, 20$. In our case duration of a lesson was $T = 300$ CUT, and control time after the end of training $t' = 350$ CUT. Also we will calculate the indicator of efficiency $K = Z / N$ which equal to the relation of amount of knowledge $Z$ at the moment $T + t'$ to total number $N$ of the studied ELM-s.

Results of modeling are given in fig. 2.3. It is visible that with growth $N$ from 3 to 21 average coefficients of forgetting $\gamma$ grows from $2 \cdot 10^{-17}$ to $4 \cdot 10^{-5}$. That is ELM-s are on average acquired worse, are forgotten quicker. At $N < 12$ amount of knowledge of the pupil $Z$ through time $t'$ after the end of training with growth $N$ increases, reaches a maximum at $N = 12$, and then at $N > 12$ decreases. It is explained by influence of two factors: 1) increase in number $N$ of the studied ELM-s; 2) reduction of number of appeals to each ELM during a lesson of the fixed duration $T$ and, as a result, deterioration of assimilation of knowledge (increasing of coefficient of forgetting $\gamma$). At small $N$ the indicator of efficiency $K$ is equal 1, and with growth $N$ it decreases to zero. So, exists such $N$ at which the level of knowledge $Z$ of the pupil through time $t'$ after the termination of a lesson of duration $T$ will be maximum. So that training was effective, it is necessary to find optimum value of speed of transfer of educational information $\nu = N / T$.

3. Training at several lessons. Now create model of studying of $N = 30$ ELM-s during of the three lessons lasting $T = 180$ CUT divided by breaks duration $T_n = 220$ CUT. On lesson the pupil addresses to one or to another ELM with equal probabilities. In process of growth of number of appealing $s_i$ to $i$-th ELM the spent time $\tau_i$ and coefficient of forgetting $\gamma_i$ decreases.

Program 3.

```
program Study_30_ELM_3_lessons; { Free Pascal }
{$N+}Uses crt, graph; Const N=30; dt=0.003; Mt=0.5;
Var t,g,tt,SZ: single; i,j,Gd,Gm: integer;
s: array[1..N] of integer; Z: array[1..N] of single;
BEGIN Gd:= Detect; InitGraph(Gd,Gm,'c:\bp\bgi');
Randomize; t:= -20; j:=1;
Repeat t:=t+dt; tt:=tt - dt;
If(tt<=0)and(((t>0)and(t<180))or((t>400)and(t<580))or ((t>800)and(t<980))) then begin j:=round(random*N);
If j>N then j:=N; If j<1 then j:=1; inc(j); If j>N then j:=1;
Z[j]:=1; inc(s[j]);
end;
For i:=1 to N do Z[i]:=2E-3*exp(-s[i]/1.5)*Z[i];
170-round(5*j),1); circle(20+round(Mt*t),170,1); circle(20+round(Mt*t),500-round(10*SZ),1); circle(20+round(Mt*t),500,1);
until KeyPressed; CloseGraph;
END.
```
FIGURE 3. SIMULATION RESULTS: THE STUDY 30 AND 60 ELM-S ON THREE LESSONS.

The program 3 is used, results of imitating modeling are given on fig. 3.1. It is visible that during training number $i$ of the considered ELM changes incidentally from 1 to 30 and the level of knowledge $Z$ of the pupil increases. During breaks pupil is forgetting, $Z$ decreases. On fig. 3.2 the result of imitating modeling of studying 60 ELM-s during three lessons of the same duration is presented. It is visible that the level of knowledge after training became much higher, but assimilation strength (coefficient of learning) significantly smaller. Because the pupil addresses to each ELM smaller number of times, they are acquired worse and after the end of training are quickly forgotten.

This computer model consider that if the number of appealing to this ELM increase then: 1) time $\tau$ of the using this ELM is reduced, tending to some limit; 2) the coefficient of forgetting $\gamma_i$ decreases, tending to zero. This model allows to simulate: 1) learning as a result of repetition of one ELM on the lesson; 2) learning as a result of repeating the many ELM-s on one lesson; 3) training as a result of repeating the many ELM-s on a few lessons. It is shown that if the rate of supply of educational information is increasing then the level of knowledge of the student through the time $t'$ after graduation, first increases, reaches a maximum and then decreases.

6. Two-component probabilistic model of the course. In the study of didactic systems typically assume that all issues (ELM-s) have the same didactic complexity and forgets equally quickly (Ivashkin, Nazojkin, 2011; Solovov, Men’shikov, 2001). Psychologists have found that it is not so: the ELM to which the student is drawn repeatedly (with which the pupil works many times), remember more firmly and forget is significantly slower than the ELM, which are practically not used. Therefore, we assume that the knowledge of a student consists of two components having different coefficient of learning and the rate of forgetting [4–6]. The proposed approach is a two-component model of knowledge according to which all perceived by the student information can be divided into two categories: 1) weak or quickly forgettable knowledge; 2) strong and slowly forgettable knowledge (skills), which are formed as a result of use of this ELM in the activities of the student.

Let the pupils study the totality of $N$ ELM-s. The level $Z_i$ of knowledge $i$-th ELM consists of: 1) the level of weak knowledge $z_i$ that have a high rate of forgetting; 2) the level established knowledge or skills $n_i$ that are slowly forgotten. You can record that $Z_i = z_i + n_i$. In addition, we assume that when the student work with $i$-th ELM, the level of knowledge increases to 1, and a part of weak knowledge become solid knowledge (or skills). Simultaneously there is a reduction of the strong and weak knowledge of all other ELM-s because forgetting.
The state of the didactic system at each moment of time is determined by two one-dimensional matrices \( z_i \) and \( n_i \) (\( i = 1, 2, ..., N \)), the elements of which lie in the interval from 0 to 1. If \( i \)-th question assimilated by the student, then \( Z_i = z_i + n_i = 1 \); but if not learned at all, then \( Z_i = z_i + n_i = 0 \). As a result of working with \( i \)-th ELM its level of assimilation increases to \( Z_i = 1 \), values \( z_i \) and \( n_i \) grows too. Almost immediately after the end of the student's work with \( i \)-th ELM starts forgetting. Over time the magnitude \( z_i \) and \( n_i \) decrease exponentially.

From psychology we know that the more one knows, the easier he is learning and remembering new information. It is logical to assume that if total knowledge of the pupil of the learning material \( S_{2n} \) and the level of knowledge of \( i \)-th ELM \( Z_i = z_i + n_i \) are the more, then the absorption coefficient \( \alpha_i \) of this ELM is the higher. When the student is working with \( i \)-th ELM (for example, the decision of simple tasks, the translation of words or sentences) he spends time \( \Delta t_i \) which at increasing the level \( Z_i \) of assimilation \( i \)-th ELM is reduced to a certain limit. We can write equations:

\[
dn_i / dt = \alpha_i (1 - n_i), \quad z_i = 1 - n_i, \quad \text{(assimilation)}
\]

\[
dz_i / dt = -\gamma z_i, \quad dn_i / dt = -\gamma n_i, \quad \text{(forgetting)}
\]

\[
n_i^{t+1} = n_i^t + \alpha_i (1 - n_i^t) \Delta t, \quad z_i^{t+1} = 1 - n_i^{t+1},
\]

\[
z_i^{t+1} = z_i^t - \gamma z_i^t \Delta t, \quad n_i^{t+1} = n_i^t - \gamma n_i^t \Delta t.
\]

The coefficient of absorption and time of working for \( i \)-th ELM:

\[
\alpha_i = a_i (a_2 + z_i + n_i - \exp(-S_{2n} / a_3)), \quad \Delta t_i = b_1 / (z_i + n_i + b_2),
\]

The total quantity of knowledge and skills is given by:

\[
S_{2n} = \sum_{i=1}^{N} Z_i = \sum_{i=1}^{N} (z_i + n_i), \quad S_n = \sum_{i=1}^{N} n_i.
\]

Coefficients are selected so that the model corresponded to real educational process. We assume that: \( a_i = 0.16 \) CUT\(^{-1} \), \( a_2 = 1.3 \), \( a_3 = 100 \), \( b_1 = 0.1 \) CUT, \( b_2 = 0.02 \), \( \gamma_z = 10^{-3} \) CUT\(^{-1} \), \( \gamma_n = 5 \cdot 10^{-5} \) CUT\(^{-1} \).

7. The determination of the parameters of the educational discipline. For example, the educational course contains 400 elements of the learning material (ELM-s) and consists of 4 themes, each of which includes 100 ELM-s. After studying of each themes follows the break which lasting 200 CUT. The extent of studying of the course is determined by the amount \( j \) previously studied ELM-s. The learning material contains both new (or planned) ELM-s and previously studied ELM-s, which are essential for understanding new topics, solve problems, perform tasks. For example, studying a foreign language, the student works with text that contains new words and words studied in the previous lessons. With the passage of the \( n \)-th theme the student is in each moment of time working with one ELM; he either meets with new issues from \( n \)-th themes, or works with a previously-studied issue of the \( n \)-th or the previous themes. We denote the probability of studying a new subject \( n \)-th theme through \( p_{new}^{\text{new}} \), and the probability of repetition (repeated requests) to the already-studied ELM-s of the \( m \)-th theme (\( m \leq n \)) through \( p_{am} \). For any disciplines, in principle, we can construct a table (Fig. 4.1), which indicates the probability of appealing of the student to the themes which study now and to the previous ones. This table will reflect the connection between the themes of
The student must work with one or another ELM, the sum of probabilities in each row of the table is equal to 1:

\[ p_{21}^{\text{new}} + p_{21} + p_{22} = 1, \quad p_{31}^{\text{new}} + p_{31} + p_{32} + p_{33} = 1, \]
\[ p_{11}^{\text{new}} + p_{11} = 1, \quad p_{41}^{\text{new}} + p_{41} + p_{42} + p_{43} + p_{44} = 1. \]

This table can be represented as a stochastic matrix (Fig. 4.2), which takes into account the degree of communication \( n \)-th and \( m \)-th themes (\( n > m \)). The higher the probability \( p_{nm} \), the more often in the study of the \( n \)-th theme the student is forced to appeal to some ELMs from the previous \( m \)-th theme of the course. Whenever you access \( i \)-th ELM, the student increases their knowledge \( z_i \) to 1, and some part of weak knowledge becomes strong (solid) knowledge.

8. The results of the simulation. For simulation of the analyzed didactic system used a computer program 4. It contains the cycle time in which: 1) using a random number generator and a given matrix of probabilities selects number \( s \) of “studied” ELM-s; 2) based on the levels of assimilation \( s \)-th ELM (variables \( z_s \) and \( n_s \)), calculated the coefficient of learning of the student \( \alpha_s \) and time of work \( \Delta t_s \) with this ELM (operators \( \alpha:=0.16*(1.3+z_s+n_s-\exp(-z_s/100)) \) and \( dt:=abs(0.1/(z_s+n_s+0.02)) \)); 3) determined the levels of knowledge of \( s \)-th ELM after the study (\( n_s:=n_s+a*(1-n_s) \) and \( z_s:=1-n_s \)); 4) calculated the levels of knowledge of all other ELM-s which wasn’t studying at this moment, these values are reduced because of forgetting (\( z_i:=(1-gz*dt)*z_i \) and \( n_i:=(1-gn*dt)*n_i \)); 5) the calculation results are displayed in graphical form. Then everything is repeated again. When a variable \( \text{obuch} \) is equal 1, the computer simulates training (begins at the moments 0, \( t_1 \), \( t_2 \), \( t_3 \)); \( \text{obuch} = 0 \) is corresponding to break duration 200 CUT.

Program 4.

\[
\begin{bmatrix}
0.32 & 0.68 & 0 & 0 & 0 \\
0.35 & 0.23 & 0.42 & 0 & 0 \\
0.43 & 0.11 & 0.27 & 0.19 & 0 \\
0.37 & 0.18 & 0.12 & 0.16 & 0.17
\end{bmatrix}
\]
s:=ff(100) else s:=100+ff(j-100); end; If (j>200)and(j<301) then begin sl:=random(100); If sl<=43 then s:=j else If sl<=54 then s:=ff(100) else If sl<=81 then s:=100+ff(100) else s:=200+ff(100) else s:=300+ff(j-300); end; a:=0.16*(1.3+z[s]+n[s]) -exp(-5z/100); dt:=abs(0.1/(z[s]+n[s]+0.02)); n[s]:=n[s]+a*(1-n[s]); z[s]:=1-n[s]; If s=j then inc(j); end; t:=t+dt; For i:=1 to M do [forgetting] If i<>s then begin z[i]:=(1-gz*dt)*z[i]; n[i]:=(1-gn*dt)*n[i]; end; SZ:=0; For i:=1 to M do SZ:=SZ+z[i]+n[i]; S1:=0; S2:=0; S3:=0; S4:=0; For i:=1 to M do begin If i<101 then S1:=S1+z[i]+n[i]; If (i>100)and(i<201) then S2:=S2+z[i]+n[i]; If (i>200)and(i<301) then S3:=S3+z[i]+n[i]; If (i>300)and(i<401) then S4:=S4+z[i]+n[i]; end; If obuch=1 then circle(round(Mt*t),500-round(Mz*SZ),1); circle(round(Mt*t),500-round(Mz*S1),1); circle(round(Mt*t),500-round(Mz*S2),1); circle(round(Mt*t),500-round(Mz*S3),1); circle(round(Mt*t),500-round(Mz*S4),1); circle(round(Mt*t),150-round(Mz*SN),1); circle(round(Mt*t),150-round(Mz*SN),1); circle(round(Mt*t),500-round(Mz*SN),1); until (KeyPressed)or(j>M)or(t>8000); For i:=1 to M do begin If i<101 then S1:=S1+z[i]+n[i]; If (i>100)and(i<201) then S2:=S2+z[i]+n[i]; If (i>200)and(i<301) then S3:=S3+z[i]+n[i]; If (i>300)and(i<401) then S4:=S4+z[i]+n[i]; end; ReadKey; CloseGraph; writeln(S1:2:3,' ',S2:2:3,' ',S3:2:3,' ',S4:2:3); END.

FIGURE 5. THE RESULTS OF A SIMULATION OF THE DISCIPLINE.

In fig. 5 shows: 1) the graphs of the dependences on time knowledge $Z_{50}$ by the student of one ELM with $i=50$ (fig. 5.1); 2) the graphs of the dependences on time total level of knowledge $S_{zn}$ and level of strong knowledge (skills) $S_n$ for each theme and for all themes of this discipline as a whole (fig. 5.2). Points in fig. 5.2 correspond to the
numbers of the studied ELM-s, which are randomly selected on the basis of the stochastic matrix in fig. 4.2. It is seen that during lessons total level of the student’s knowledge $S_{zn}$ and the level of formation of skill $S_n$ increase; during breaks and after the training they decline as a result of forgetting. In this time weak knowledge is quickly forgotten, only remained strong knowledge. The upper graph (fig. 5.1) shows how in certain moments of time $t_1', t_2', t_3', t_4'$ (when the student turns to the 50th ELM), there is a rapid increase of the level of knowledge $Z_{50}$ to 1; after that there is exponential decay. In these moments, the level of formation of skill $n_{50}$ is fast increasing; after end of working with 50th ELM $n_{50}$ decreases very slowly.

The proposed simulation model of the discipline takes into account: 1) connections between course topics, using the probability of appealing to ELM-s of the previous themes; 2) the increase in number of appeals of the pupil to ELM-s leads to growing of a share of strong knowledge; 3) reduction of working time with the ELM because of increasing the number of it usages; 4) increase the coefficient of learning for a given ELM when the total amount of knowledge and/or knowledge of this ELM is growing. This approach allows to trace the dynamics of the student’s total knowledge (strong and weak) and knowledge of each theme.

**9. Multi-component continuous model of learning.** We will designate through $U$ the level of requirements of teacher. It equal to quantity $Z_0$ of ELM-s which are transmitted to pupils. Let $Z$ - total knowledge of the pupil which include knowledge of the first, second and third categories: $Z = Z_1 + Z_2 + Z_3$. Thus $Z_1$ - quantity of the most weak knowledge $K_1$ of the first category with high coefficient of forgetting $\gamma_1$, and $Z_3$ - quantity of the strongest knowledge $K_3$ of the third category with low $\gamma_3$ ($\gamma_3 < \gamma_2 < \gamma_1$). Coefficients of assimilation $\alpha_i$ characterize speed of transition of knowledge $(i - 1)$-th categories in knowledge $i$-th categories. The offered three-component model of training is expressed by system of the equations:

$$
\frac{dZ_1}{dt} = k\alpha_1(U - Z)Z^b - k\alpha_2Z_1 - \gamma_1Z_1, \quad \frac{dZ_2}{dt} = k\alpha_2Z_1 - k\alpha_3Z_2 - \gamma_2Z_2, \\
\frac{dZ_3}{dt} = k\alpha_3Z_2 - \gamma_3Z_3, \quad Z = Z_1 + Z_2 + Z_3.
$$

While there is a training ($k = 1$), the speed of increase of weak knowledge of the pupil is proportional: 1) differences between level of requirements of the teacher $U$ and the general level of knowledge $Z$; 2) to amount of already available knowledge $Z$ in degree $b$. The last is explained by that existing knowledge promotes establishment of new associative links and memorizing of new information. If the increase of knowledge of pupil is significantly less than their total quantity then $b = 0$. When training stops ($k = 0$), $Z$ decreases due to forgetting. The coefficient of forgetting $\gamma_i = 1/\tau_i$, where $\tau_i$ - time during which the amount of knowledge $i$-th categories decreases in $e = 2,72...$ times. The result of training is characterized by a total quantity of the acquired knowledge $Z = Z_1 + Z_2 + Z_3$ and coefficient of strength $P = (Z_2 / (2 + Z_3)) / Z$. If all knowledge acquired during training is weak ($Z_1 = Z$, $Z_2 = Z_3 = 0$), then coefficient of strength $P = 0$. It is necessary to aspire to a situation, when all acquired knowledge strong ($Z_3 = Z$, $Z_1 = Z_2 = 0$), then $P = 1$. With long-term learning of one theme the quantity of knowledge $Z$ increases to $U$, also there is an increase of quantity of strong knowledge $Z_4$ and coefficient of strength $P$. 
10. Using of the model in the analysis of concrete situations. Let analyze some situations which are arising in the pedagogical practice.

Situation 1. The teacher conducts three lessons, level of requirements during each lesson is set: $U_1$, $U_2$, $U_3$. We will study process of training of the pupil by means of four-component model. Results of modeling are presented in fig. 6.1. It is visible that during training the total of knowledge $Z$ of the pupil increases, part of weak knowledge become stronger. During breaks and after training level of weak knowledge $Z_1$ quickly decreases, and strong knowledge $Z_4$ is forgotten significantly more slowly.

Situation 2. The teacher conducts three lessons, level of requirements $U(i)$ during $i$-th lesson increases in accordance with the equations $U_i = a_i(t_i - t_0) + b_i$, $i = 1, 2, 3$. Let analyze training process by means of two-component model which is expressed by the equations:

$$\frac{dZ_1}{dt} = k\alpha_1(U - Z) - k\alpha_2Z_1 - \gamma_1Z_1,$$

$$\frac{dZ_2}{dt} = k\alpha_2Z_1 - \gamma_2Z_2, \quad Z = Z_1 + Z_2. $$

Results of modeling are given in fig. 6.2. At each lesson the teacher demands from pupils: 1) possession of the material studied at the previous lessons; 2) assimilation of new information. During training weak knowledge becomes strong (solid) and after training is forgotten significantly more slowly.

Situation 3. The pupil within 11 years studies at school. The coefficient of assimilation of information during of training increases and set by a matrix $\alpha_j = (0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.045, 0.05, 0.055, 0.06)$. All educational information is divided into two categories: 1) knowledge and skills $K_1$ which are used daily and therefore are badly forgotten (reading, writing, arithmetic actions, the simple facts, etc.); 2) knowledge $K_2$ which are applied seldom and therefore are quickly forgotten (difficult theories, principles, lows, facts). Levels of requirements the teachers corresponding to knowledge $K_1$ and $K_2$ which need to be acquired in $j$-th class, are set by matrixes: $U_{1j} = (50, 46, 42, 36, 30, 25, 20, 15, 10, 10, 10)$ and $U_{2j} = (4, 8, 14, 18, 24, 28, 33, 38, 46, 58, 62)$. Coefficients of forgetting of knowledge $K_1$ and $K_2$ are equal $\gamma_1 = 0.002$ and $\gamma_2 = 0.01$ consequently. It is necessary to calculate a total level of knowledge and amount of knowledge $K_1$ and $K_2$ at various moments $t$.
In this case knowledge $K_1$ and $K_2$ has different speed of forgetting, but don't pass from one category into another. The offered two-component model of training is expressed by system of the equations:

$$dZ_1/dt = k\alpha_1(U_1 - Z_1)Z_1^b - \gamma_1Z_1,$$

$$dZ_2/dt = k\alpha_2(U_2 - Z_2)Z_2^b - \gamma_2Z_2, \quad Z = Z_1 + Z_2.$$

Here $U_1$ and $U_2$ - levels of requirements the teachers corresponding to knowledge $K_1$ and $K_2$ which quantity is equal $Z_1$ and $Z_2$, and $Z$ - total knowledge of the pupil.

Results of use of computer model are presented in fig. 7. On it are shows: 1) graphics $Z_1(t)$ and $Z_2(t)$ dependences of knowledge $K_1$ and $K_2$ from time; 2) the curve of dependence of total of knowledge $Z$ from time; 3) graphics $Z_1'(t)$ and $Z_2'(t)$ dependences of knowledge $K_1$ and $K_2$ acquired by the pupil in the 10-th class from time. It is visible that during training in school the total amount of knowledge, and also quantity of knowledge $K_1$ and $K_2$ monotonously increase, and after training decrease due to forgetting. Knowledge $K_1$ is forgotten significantly quicker, than $K_2$. Values of the parameters are picked up so that the model corresponded to the typical situation meeting in student teaching.

![Graph](image)

**FIGURE 7. CHANGE OF AMOUNT OF KNOWLEDGE AT STUDYING WHEN TRAINING AT SCHOOL.**

9. Conclusion. The main objective of the mathematical theory of teaching consists in the following: based on the parameters of the students (coefficients of absorption, forgetting, etc.), characteristics of the used methods and a curriculum that specifies the distribution of educational information, one needs to determine the level of knowledge of students in the learning process and after its completion. The advantage of using simulation models at the analysis of the system “teacher-student” is in comprehensive enumeration and consideration of all factors affecting on its behavior. The results of computer simulations of the learning process complements qualitative reasoning, enhances their objectivity and validity. This method is useful if the conduct of the pedagogical experiment can give a negative result or expensive. Changing the sequence of the various ELM-s, length of lessons, etc., you can use the computer model to find the optimal way of learning in a particular case.
This paper develops the ideas presented by the author in monograph (Mayer, 2014), which also analyzed: 1) discrete and continuous one-component model of learning; 2) strategies of interaction between teacher and student; 3) multi-component model of learning; 4) takes into account changes in the workability of the student; 5) the search for optimal ways of learning by using discrete and continuous models; 6) simulation of learning of the themes and issues associated genetic relationship; 7) the agreement between the results of simulation of the learning process with the test results. The use of this computer models allows you to create a digital model of a disciple, to prove different regularities of educational process, to study the dependence of the result of learning from the educational characteristics of the material, students, duration and number of lessons, etc.

One of the directions of using of such models is to create a training program that simulates the learning process in the school, which is designed to train students of pedagogical universities. It should allow changing the parameters of the disciples, the duration of training, distribution of educational material and strategies of teacher interaction with pupils. In the course of its work the student playing the role of the teacher, changes the rate of to speeds of transfer of educational information and responds quickly to the questions of students, conducts tests, puts marks, trying to achieve the greatest level of knowledge in a given time. After “learning” the computer displays graphs showing the “knowledge of pupils”, evaluation “completed tests”, etc. In addition, the training program can analyze the work of system “teacher-pupil” and assessing student’s activity.

REFERENCES